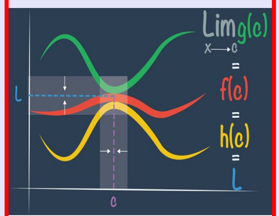
Calculus I

Lecture 7



Feb 19-8:47 AM

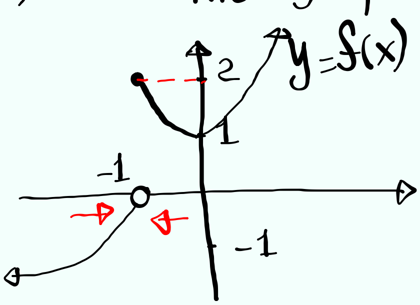
Class Quiz 2

Box Your Final Ans.

1) Evaluate $\lim\limits_{x \to 2} \dfrac{x^2-3x+2}{x^2-4} = \dfrac{0}{0}$ I.F.

$= \lim\limits_{x \to 2} \dfrac{\cancel{(x-2)}(x-1)}{\cancel{(x-2)}(x+2)} = \lim\limits_{x \to 2} \dfrac{x-1}{x+2} = \dfrac{2-1}{2+2} = \boxed{\dfrac{1}{4}}$

2) Use the graph of $y=f(x)$ below to find



a) $\lim\limits_{x \to -1^-} f(x) = \boxed{0}$

b) $\lim\limits_{x \to -1^+} f(x) = \boxed{2}$

Sep 5-7:08 AM

More on limits

1) Evaluate $\lim_{h\to 0} \frac{(5+h)^2-25}{h} = \frac{0}{0}$

$$= \lim_{h\to 0} \frac{\cancel{25}+10h+h^2\cancel{-25}}{h} = \lim_{h\to 0} \frac{\cancel{h}(10+h)}{\cancel{h}} = \lim_{h\to 0}(10+h)$$

$$= \boxed{10}$$

Sep 5-7:38 AM

2) Evaluate $\lim_{x\to 16} \frac{4-\sqrt{x}}{16x-x^2} = \frac{4-\sqrt{16}}{16(16)-16^2} = \frac{4-4}{16^2-16^2} = \frac{0}{0}$

$A - B \quad A+B \qquad A^2 - B^2 \quad$ I.F.

$$\lim_{x\to 16} \frac{(4-\sqrt{x})\cdot(4+\sqrt{x})}{x(16-x)\cdot(4+\sqrt{x})} = \lim_{x\to 16} \frac{\overset{1}{\cancel{16-x}}}{x\cancel{(16-x)}(4+\sqrt{x})}$$

$$= \lim_{x\to 16} \frac{1}{x(4+\sqrt{x})} = \frac{1}{16(4+\sqrt{16})}$$

$\frac{a-b}{b-a} = -1$

$\frac{5-3}{3-5} = \frac{2}{-2} = -1$

$$= \frac{1}{16\cdot 8} = \boxed{\frac{1}{128}}$$

Sep 5-7:44 AM

3) Evaluate $\lim_{h\to 0} \dfrac{\dfrac{1}{(x+h)^2} - \dfrac{1}{x^2}}{h} = \dfrac{0}{0}$ I.F.

$LCD = (x+h)^2 \cdot x^2$

$$\lim_{h\to 0} \frac{\cancel{(x+h)^2} \cdot x^2 \cdot \dfrac{1}{\cancel{(x+h)^2}} - (x+h)^2 \cdot \cancel{x^2} \cdot \dfrac{1}{\cancel{x^2}}}{h\,(x+h)^2 \cdot x^2}$$

$$= \lim_{h\to 0} \frac{x^2 - (x+h)^2}{h(x+h)^2 \cdot x^2} = \lim_{h\to 0} \frac{\overbrace{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}^{\cancel{h}(-2x-h)}}{\cancel{h}(x+h)^2 \cdot x^2}$$

$$= \lim_{h\to 0} \frac{-2x-h}{(x+h)^2 \cdot x^2} = \frac{-2x}{x^2 \cdot x^2} = \boxed{\frac{-2}{x^3}}$$

Sep 5-7:51 AM

1) Simplify $\dfrac{1}{x\sqrt{x+1}} - \dfrac{1}{x}$

$$= \frac{1}{x\sqrt{x+1}} - \frac{1\sqrt{x+1}}{x\sqrt{x+1}} = \frac{1-\sqrt{x+1}}{x\sqrt{x+1}}$$

2) $\lim_{x\to 0} \left[\dfrac{1}{x\sqrt{x+1}} - \dfrac{1}{x} \right] = \lim_{x\to 0} \dfrac{1-\sqrt{x+1}}{x\sqrt{x+1}} = \dfrac{0}{0}$ I.F.

$$= \lim_{x\to 0} \frac{(1-\sqrt{x+1})(1+\sqrt{x+1})}{x\sqrt{x+1}\,(1+\sqrt{x+1})}$$

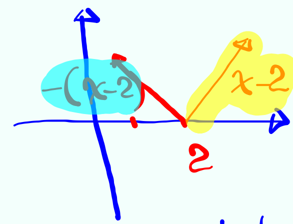
$$= \lim_{x\to 0} \frac{(1)^2 - (\sqrt{x+1})^2}{x\sqrt{x+1}\,(1+\sqrt{x+1})} = \lim_{x\to 0} \frac{-\cancel{x}}{\cancel{x}\sqrt{x+1}(1+\sqrt{x+1})}$$

$$= \lim_{x\to 0} \frac{-1}{\sqrt{x+1}(1+\sqrt{x+1})} = \frac{-1}{1(1+1)} = \boxed{\frac{-1}{2}}$$

Sep 5-7:58 AM

Evaluate $\lim_{x \to 2^+} \dfrac{x^2+x-6}{|x-2|}$

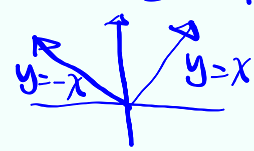
$y=|x-2|$



$= \lim_{x \to 2^+} \dfrac{\overbrace{x^2+x-6}^{\cancel{(x-2)}(x+3)}}{\cancel{x-2}} = \boxed{5}$

Evaluate $\lim_{x \to 2^-} \dfrac{x^2+x-6}{|x-2|}$

$y=|x|$



$= \lim_{x \to 2^-} \dfrac{\cancel{(x-2)}(x+3)}{-\cancel{(x-2)}} = \boxed{-5}$

Sep 5-8:06 AM

Evaluate $\lim_{x \to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1} = \dfrac{\sqrt[3]{1}-1}{\sqrt{1}-1} = \dfrac{0}{0}$

For numerator, we must use A^3-B^3 Factor.

$\lim_{x \to 1} \dfrac{(\sqrt[3]{x}-1)(\sqrt{x}+1)(\;^2+\;+\;^2)}{(\sqrt{x}-1)(\sqrt{x}+1)(\;^2+\;+\;^2)}$

Let $x = u^6$

as $u \to 1$, $x \to 1$

$\lim_{x \to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt{x}-1} =$

$\lim_{u \to 1} \dfrac{\sqrt[3]{u^6}-1}{\sqrt{u^6}-1} =$

$= \lim_{u \to 1} \dfrac{u^2-1}{u^3-1}$ $\quad \dfrac{0}{0}$

$= \lim_{u \to 1} \dfrac{\cancel{(u-1)}(u+1)}{\cancel{(u-1)}(u^2+u+1)} = \boxed{\dfrac{2}{3}}$

Sep 5-8:12 AM

Find a & b such that

$$\lim_{x \to 0} \frac{\sqrt{ax+b} - 2}{x} = 1 \checkmark$$

$$\lim_{x \to 0} \frac{(\sqrt{ax+b} - 2)(\sqrt{ax+b} + 2)}{x(\sqrt{ax+b} + 2)}$$

$\to 0 \to \boxed{b=4}$

$$= \lim_{x \to 0} \frac{ax + b - 4}{x(\sqrt{ax+b} + 2)} = \lim_{x \to 0} \frac{a\cancel{x}}{\cancel{x}(\sqrt{ax+4} + 2)}$$

$$= \lim_{x \to 0} \frac{a}{\sqrt{ax+4} + 2} = \frac{a}{\sqrt{4}+2} = \frac{a}{4} = 1$$

$\boxed{b=4}$ $\boxed{a=4}$

1) Look for my email this weekend.

2) Google limit laws.

$$\lim_{x \to a} [f(x) + g(x)]$$

$$\lim_{x \to a} [f(x) - g(x)]$$

$\vdots$

Sep 5-8:24 AM